parameter = numerical characteristic of a population (σ, μ, p)
statistic = " " " " sample (s, $\bar{x}$, $\hat{p}$)

BIRZEIT UNIVERSITY

the sample statistics are point estimators of the population parameters.

MATHEMATICS DEPARTMENT

Final Exam Stat 236 Spring 2006

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Question # 1(15%)

30 % of all households in Ramallah have ADSL Internet access. In a sample of 20 houses, let X be the number of households in the sample who have ADSL.

1. How X is distributed. Find the expected value and the variance of X.
2. Find the probability that between 5 and 12 of the households have an ADSL.
3. Find the probability that at least 10 out of the twenty households have an ADSL.

$n=20 \quad p=30\%$

(1) $X \sim$ binomial distribution

$E(\bar{x}) = \mu = np = 20 \times 30\% = 6$

$Var(x) = np(1-p) = 20 \times 30\% \, (1-30\%) = 4.2$

Standard deviation $= \sqrt{4.2} = 2.04$

(2) $p(5 < x < 12)$

$n = 20$

$np = 20 \times 30\% = 6 > 5$

$n(1-p) = 20(1-30\%) = 14 > 5$

So we can app Binomial by normal.

$p\left(\frac{5-6}{2.04} < z < \frac{12-6}{2.04}\right) = p(-0.49 < z < 2.94)$

(3). $p(X \geq 10) = p(X > 9.5)$

$= p\left(z > \frac{9.5-6}{2.04}\right) = p(z > 1.71)$

poisson

Question # 2(10%)

The average number of cars entering a roundabout is 5 cars per minute. Cars arrive randomly and independently.

1. What is the probability that 3 or more cars will arrive at the roundabout in the next minute.
2. What is the probability that 3 or more cars will arrive at the roundabout in the next 3 minutes.

(1). $\mu = 5$ per minute

$$p(X \geq 3) = 1 - (p(X=2) + p(X=1) + p(X=0))$$

$$= 1 - \left(\frac{e^{-5} \cdot (5)^2}{2!} + \frac{e^{-5} \cdot (5)^1}{1!} + \frac{e^{-5} \cdot (5)^0}{0!} \right)$$

(2) $\mu = 5 \longrightarrow$ 1min

$? \longleftarrow$ 3min

$$\frac{3 \times 5}{1} = 15 = \mu$$

$$p(X \geq 3) = 1 - (p(X=2) + p(X=1) + p(X=0))$$

$$= 1 - \left(\frac{e^{-15} \cdot (15)^2}{2!} + \frac{e^{-15} \cdot (15)^1}{1!} + \frac{e^{-15} \cdot (15)^0}{0!} \right)$$

Question #3(15%)

The populations mean household income in a city is \$50000and a standard deviation is \$18000. The city has 5000 households. A simple random sample of size 324 is selected,

1. Find expectation of the sample mean. Find the standard error of the sample mean.
2. What is the probability that the sample mean will be more than \$52500?
3. Is it likely that the average income in the sample will be less than \$46500? Explain.

$\bar{X} \sim$ normal $\quad n > 30 \rightarrow$ C.L.T

Zscore

(-3, 3)

$E(\bar{X}) = \mu$

$\mu = 50{,}000 \qquad N = 5000$

$\sigma_{\bar{x}} \qquad \sigma = 18000 \qquad n = 324$

E

(a) $E(\bar{X}) = \mu = 50{,}000$

$\frac{n}{N} = \frac{324}{5000} = 0.0648 > 0.05$

So the pop is finite

$\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}} \cdot \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{5000-324}{5000-1}} \cdot \frac{18000}{\sqrt{324}}$

$= \sqrt{0.9352} \cdot 1000$

$= 967.05$

(b). $p(\bar{X} > 52500) = p\left(z > \frac{52500 - 50{,}000}{967.05}\right)$

(c). $p(\bar{X} < 46500) = p(z < -3.61)$

$= 0$

It's not likely.

$\hat{p} = 50\%$

Question # 4(15%)

You want to construct a confidence interval for the percent of registered Palestinian voters who are planning on voting for Abu Mazen for president for his second Term. You want to have a margin of error of ± .03

1. How many registered voters should you survey. Use the 95% confidence level
2. Suppose that you conducted this survey (as in part 1) and found 555 of the respondents intended to vote for Abu Mazen. Construct the appropriate 99% confidence interval. Interpret this interval.
3. Based on your interval in part (2), is it reasonable to conclude that the majority (more than half) of the population intended to vote for Abu Mazen? Explain

$n = 555$

$\alpha = 0.01$

$\hat{p} \mp Z_{\alpha/2}\,\sigma_{\hat{p}}$

[48%, 52%]

$\pm 0.03 \quad p = 50\%$

$n = \frac{(Z_{\alpha/2})^2\, p(1-p)}{E^2} = \frac{(1.96)^2\, 50\%(1-50\%)}{(0.03)^2}$

$= 1067.11 \approx 1067$

90% $Z_{\alpha/2} = 1.645$

95% " 1.96

99% " 2.545

$\hat{p} \mp Z_{\alpha/2}\,\sigma_{\hat{p}}$

$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

$[50 \mp (2.545)(0.001)]$

$\sigma_{\hat{p}} = \sqrt{\frac{50\%(1-50\%)}{555}} = 0.001$

$[50 \mp 0.053]$

$[49.94, 50.05]$

because the interval is between 49.94 and 50.05

4

Question # 5(15%)

A real estate agency wants to estimate the average selling price of all houses in Ramallah. It randomly samples 25 recent sales and find that the average price is $\bar{x}$=110000 and the standard deviation is s = $30000

1. What is the population of interest in question? What is the parameter? What is the statistic?

2. Construct and interpret the 99% confidence interval for the mean of all recent selling prices. What assumption is necessary to construct this interval?

→ normal

3. We recently heard that a friend paid $150000 for a house in Ramallah. Can this be accepted? Explain.

iza bil interval.

$\bar{X}$ = 110,000 n=25 S= 30,000

(1). population = all houses in Ramallah

parameter: pop mean (μ)

statistic = sample mean ($\bar{x}$)

(2). 99% $z_{\alpha/2}$ = 2.575

$\bar{X} \pm z_{\alpha/2} \cdot S/\sqrt{n}$

[110,000 ± (2.575 (30,000/$\sqrt{25}$))]

[110,000 ∓ 15270]

[94730, 125270]

(3). No it can't be accepted because 15000 is not within the confidence interval

Question # 6(15%)

Small increases in the mean charge for monthly long- distance telephone calls produce substantial in the profits for telephone companies. A telephone company's records indicate the private customers pay an average of $20 per month for long-distance telephone calls; the standard deviation of the amount paid for long-distance calls is $15.

1. If a random sample of 100 bills is taken, what is the probability that a sample mean is greater than 23?

2. A random sample of 100 customer's bill during a given month produced a sample mean of $22.25 expended for long-distance calls. Do the data indicate that the mean level of the amounts billed per month for long-distance telephone calls has changed? Test using $\alpha = 0.01$

3. Construct a 99% confidence interval for the population mean. What is your conclusion depending on this interval?

$\sigma = 15 \quad \mu_0 = 20$

$n = 100$

$H_0: \mu = \mu_0 = 20$

$H_a: \mu > 20$

$H_0: \mu = 20$

$H_a: \mu > 20$

(1). $p(\bar{X} > 23) \quad n = 100 \quad \mu = 20 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{100}} = \frac{15}{10} = 1.5.$

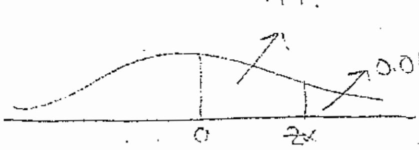
$p\left(\frac{\bar{X}-\mu}{\sigma} > \frac{23-20}{1.5}\right) = p(z > 2)$

(2). $H_0: \mu = \mu_0 = 20$ $\quad \bar{X} = 22.25 \quad n = 100. \quad \alpha = 0.01$

$H_a: \mu > 20$

$z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{22.5 - 20}{15/\sqrt{100}} = 1.66$

49?



$z_\alpha = 2.33$

$z = 1.66 < z_\alpha = 2.33 \Rightarrow$ accept H_0

(3). 99% $z_{\alpha/2} = 2.545$

$\bar{x} \pm z_{\alpha/2}\, \sigma/\sqrt{n}$

6

$[22.5 \mp (2.545) \cdot 15/\sqrt{100}] \quad [22.5 \pm 3.8127$

18.6825

accept H_0 [illegible]

Question # 7(15%)

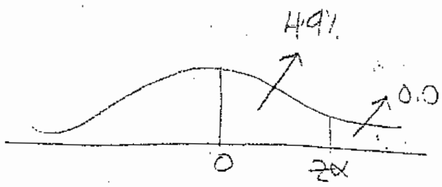
The average weekly earnings for all full time equivalent employees are reported to be \$345. Suppose that you want to check this claim since you believe it is too low. You want to prove that the average weekly earnings of all employees are higher than the amount stated. You collect a random sample of 900 employees in all areas and find that the sample mean is \$360 and the sample standard deviation is \$135. Can you disprove the claim? Use $\alpha = 1\%$

$n = 900 \quad \bar{X} = 360 \quad S = 135 \quad \alpha = 0.01$

$H_0: \mu = \mu_0 = 345$

$H_a: \mu > 345$

$$z = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{360 - 345}{135/\sqrt{900}} = \frac{15}{4.5} = 3.33$$



$z_\alpha = 2.33$

z: $3.33 > z_\alpha = 2.33$, reject Ho.

yes

- Sample standard deviation

$$s=\sqrt{\frac{(x-\bar{x})^2}{n-1}},\quad s=\sqrt{\frac{\sum x^2-\frac{\sum x}{n}}{n-1}}$$

- Standardized value (z-score)

$$z=\frac{x-\mu}{\sigma}$$

- **Binomial Probability Distribution**

$$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$$

E(X) = np, $\sigma(X)=\sqrt{np(1-p)}$

Poisson Probability Distribution

$$f(x)=\frac{\mu^x e^{-\mu}}{x!}$$

- Sampling Distribution of the mean

$E(\bar{x})=\mu$, $\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}$ or $\sigma_{\bar{x}}=\sqrt{\frac{N-n}{n-1}}\frac{\sigma}{\sqrt{n}}$

- Sampling Distribution of the proportion

$E(\bar{p})=p$, $\sigma_{\bar{p}}=\sqrt{\frac{p(1-p)}{n}}$

- Confidence interval for the mean

$\bar{x}\pm z_{\frac{\alpha}{2}}\,\sigma_{\bar{x}}$, or $\bar{x}\pm t_{\frac{\alpha}{2}}\,s_{\bar{x}}$ df = n – 1, Maximum error of estimate E = $z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$

- Confidence interval for the proportion

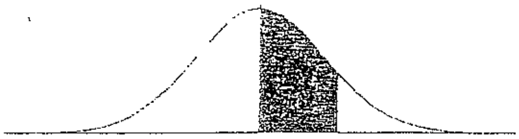
$\bar{p}\pm z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}$ Maximum error of estimate E = $z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}$

- Hypothesis test for the mean

Statistic test: (Large sample case) $z=\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$,

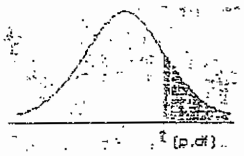
$$\frac{E^2}{z_{\frac{\alpha}{2}}\,S^2}$$

or (Small sample case) $t=\frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$ df = n-1



$P(0 \le Z \le z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990



df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460